(23) Evaluate $\underset{x \to a}{Lt} \dfrac{\log(x-a)}{\log(e^x - e^a)}$.

Ans. → $\underset{x \to a}{Lt} \dfrac{\log(x-a)}{\log(e^x - e^a)} \left[\dfrac{\infty}{\infty}\right]$

Hence, from L' Hospital's Rule

$= \underset{x \to a}{Lt} \dfrac{\frac{1}{x-a}}{\frac{1}{e^x - e^a} \times e^x} = \underset{x \to a}{Lt} \dfrac{1}{x-a} \times \dfrac{e^x - e^a}{e^x} \left[\dfrac{0}{0}\right]$

Hence, from L' Hospital Rule

$= \underset{x \to a}{Lt} \dfrac{e^x}{(x-a)e^x + e^x \times 1}$

$$= \underset{x\to a}{Lt} \frac{\cancel{e^x}}{\cancel{e^x}(x-a+1)}$$

$$= \frac{1}{\cancel{x}-\cancel{a}+1} = \frac{1}{1} = 1 \text{ Ans.}$$

(24) Evaluate $\underset{x\to 0}{Lt} \frac{\log x^2}{\log \cot^2 x}$

Ans. → $\underset{x\to 0}{Lt} \frac{\log x^2}{\log \cot^2 x} = \frac{2\log x}{2\log \cot x} \left[\frac{\infty}{\infty}\right]$

Hence from L' Hospital Rule

$$= \underset{x\to 0}{Lt} \frac{\frac{1}{x}}{\frac{1}{\cot x} \times -\operatorname{cosec}^2 x} = \underset{x\to 0}{Lt} \frac{\frac{1}{x}}{-\frac{\operatorname{cosec}^2 x}{\cot x}}$$

$$= \underset{x\to 0}{Lt} -\frac{\frac{1}{x}}{\frac{\frac{1}{\cancel{\sin^2 x}}\sin x}{\frac{\cos x}{\cancel{\sin x}}}} = \underset{x\to 0}{Lt} -\left(\frac{\tan x}{x}\right) = -1 \text{ Ans.}$$

(25) Evaluate $\underset{x\to \frac{1}{2}\pi}{Lim} \frac{\log(x-\frac{1}{2}\pi)}{\tan x}$

Ans. → $\underset{x\to \frac{1}{2}\pi}{Lt} \frac{\log(x-\frac{1}{2}\pi)}{\tan x} \left[\frac{\infty}{\infty}\right]$

Hence from L' Hospital's Rule

$$= \underset{x \to \frac{1}{2}\pi}{Lt} \frac{\frac{1}{x - \frac{1}{2}\pi}}{\sec^2 x}$$

$$= \underset{x \to \frac{1}{2}\pi}{Lt} \frac{\frac{1}{x - \frac{1}{2}\pi}}{\frac{\cancel{\sec^2 x}}{\frac{1}{\cos^2 x}}} = \frac{1}{x - \frac{1}{2}\pi} \times \frac{\cos^2 x}{1}$$

$$= \underset{x \to \frac{1}{2}\pi}{Lt} \frac{\cos^2 x}{x - \frac{1}{2}\pi} \left[\frac{0}{0}\right]$$

Hence from L' Hospital Rule,

$$= \underset{x \to \frac{1}{2}\pi}{Lt} \frac{-2\cos x \cdot \sin x}{1}$$

$$= \frac{-2\cos\frac{1}{2}\pi \cdot \sin\frac{1}{2}\pi}{1}$$

$$= \frac{-2\cos\frac{1}{2}\times 1 \cdot \sin\frac{1}{2}\times 0}{1}$$

$= \frac{0}{1} = 0$ Ans.

(26) Evaluate $\underset{x \to \infty}{Lt} \frac{x^n}{e^x}$, n being a positive integer

Ans. → $\underset{x \to \infty}{Lt} \frac{x^n}{e^x} \left[\frac{\infty}{\infty}\right]$

Hence from L' Hospital's Rule

$$\underset{x \to \infty}{Lt} \frac{n x^{n-1}}{e^x} \left[\frac{\infty}{\infty}\right]$$

$$= \underset{x \to \infty}{Lt} \frac{n(n-1)x^{n-2}}{e^x} \left[\frac{\infty}{\infty}\right]$$

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$$= \underset{x \to \infty}{Lt} \frac{n(n-1)(n-2)(n-3)\cdots 3 \cdot 2 \cdot 1 \cdot x^0}{e^x}$$

$$= \underset{x \to \infty}{Lt} \frac{\underline{|n}}{e^x} = \frac{\underline{|n}}{\infty} = 0.$$

(28) Evaluate $\lim_{n \to \infty} n \sin\frac{1}{n}$.

Ans.→ $\underset{n \to \infty}{Lt}\ n \sin\frac{1}{n} \ [\infty \times 0]$

Let $\frac{1}{n} = \theta$

$$= \underset{n \to \infty}{Lt} \frac{\sin\frac{1}{n}}{\frac{1}{n}}$$

$\therefore n \to \infty$

$\theta \to 0$

$$= \underset{n \to \infty}{Lt} \frac{\sin\theta}{\theta} = 1. A$$

(29) Determine $\lim (x \log x)$, as $x \to 0$

Ans.→ $\underset{x \to 0}{Lt}\ x \log x \ [0 \times \infty]$

$$\underset{x \to 0}{Lt} \frac{\log x}{\frac{1}{x}} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital Rule

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(\log x)}{\frac{d}{dx}\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to 0} \frac{\cancel{\frac{1}{x}}}{-\cancel{\frac{1}{x \cdot x}}} = 0 \text{ Ans.}$$

(30) Evaluate $\lim_{x\to 0} x \log \tan x$

Ans.→ $\lim_{x\to 0} x \log \tan x \quad [0 \times \infty]$

$$= \lim_{x\to 0} \frac{\log \tan x}{\frac{1}{x}} \left[\frac{\infty}{\infty}\right]$$

Hence, from L' Hospital Rule

$$= \lim_{x\to 0} \frac{\frac{\sec^2 x}{\tan x}}{-\frac{1}{x^2}} = \frac{\frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}}}{-\frac{1}{x^2}} = \frac{1}{\cancel{\cos^2 x}\, \cos x} \times \frac{\cancel{\cos x}}{\sin x} \times -\frac{x^2}{1}$$

$$= \lim_{x\to 0} \frac{-x^2}{\cos x \cdot \sin x} \left[\frac{0}{0}\right]$$

Hence from L' Hospital Rule

$$= \lim_{x\to 0} \frac{-2x}{(\cos x \cdot \cos x - \sin x \cdot \sin x)}$$

$$= \lim_{x\to 0} \frac{-2x}{\cos^2 x - \sin^2 x} = \frac{0}{1-0} = \frac{0}{1} = 0.$$